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HIGHER-SPIN-MATTER INTERACTIONS IN 2+1 DIMENSIONS

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Abstract

We describe a model of massive matter fields interacting through higher-spin gauge fields in 2+1 dimensional space-time. The two main conclusions are that the parameter of mass M appears as a module characterizing an appropriate vacuum solution of the full non-linear model and that M affects a structure of a global vacuum higher-spin symmetry which leaves invariant the chosen vacuum solution.

1 Introduction

Higher-spin (HS) symmetries are (infinite-dimensional) symmetries generated by conserved currents which contain higher derivatives of dynamical fields. HS gauge theories are most symmetric gauge theories which can underlie a fundamental theory of unified interactions. In 3+1 space-time dimensions HS gauge fields describe massless fields of arbitrarily high spin which have their own degrees of freedom [1]. In lower dimensions HS gauge fields do not propagate rather mediating interactions of matter sources analogously to the case of the gravitational field in 2+1 dimensions [2, 3]. Analysis of HS interactions of relatively simple lower dimensional models is useful since it sheds some light on general properties of HS models. In this report which is based on a recent work completed with S. Prokushkin we focus on the specificities of the HS interactions for the case of massive matter fields.

Originally it was conjectured by Blencowe [4] that a 3d HS algebra is the direct sum of two Heisenberg-Weyl algebras (more precisely, of their Lie supercommutator superalgebras). More generally, 3d HS gauge fields A may correspond to any algebra g which contains the anti-de Sitter (AdS) algebra $o(2,2) \sim sp(2) \oplus sp(2)$ as a subalgebra and admits a non-degenerate invariant bilinear form which allows one to write the Chern-Simons action for the pure gauge HS system, $S = \int_{M_3} str(A \wedge dA + \frac{2}{3} A \wedge A \wedge A)$. In [5, 6, 7] it was shown that there exists a one-parametric class of infinite-dimensional algebras which we denote $hs(2; \nu)$ (ν is an arbitrary real parameter), all containing $sp(2)$ as a subalgebra. This allows one to define a class of HS algebras $g = hs(2; \nu) \oplus hs(2; \nu)$. The supertrace operation was defined in [7] where also the following useful realization of (supersymmetric extension of) $hs(2; \nu)$ was given.

Consider an associative algebra $Aq(2, \nu)$ with a general element of the form

$$f(q, K) = \sum_{n=0}^{\infty} \sum_{A=0,1} \frac{1}{2^i n!} f^{A\alpha_1 \dots \alpha_n}(K) q_{\alpha_1} \dots q_{\alpha_n} , \quad (1)$$

under condition that the coefficients $f^{A\alpha_1 \dots \alpha_n}$ are symmetric with respect to the indices $\alpha_j = 1, 2$ while the generating elements q_α satisfy the relations

$$[q_\alpha, q_\beta] = 2i\epsilon_{\alpha\beta}(1 + \nu K); \quad Kq_\alpha = -q_\alpha K; \quad K^2 = 1; \quad \epsilon_{12} = 1. \quad (2)$$

The two important properties of this algebra are that (i) it admits the following unique supertrace operation [7] $str(f) = f^0 - \nu f^1$, such that $str(fg) = (-1)^{\pi_f \pi_g} str(gf)$, $\forall f, g$ having a definite parity, $f(-q, K) = (-1)^{\pi_f} f(q, K)$ (i.e. $str(1) = 1$, $str(K) = -\nu$ and all higher monomials of q_α in (1) do not contribute under the supertrace) and (ii) that for all ν the bilinears $T_{\alpha\beta} = \frac{1}{4i} \{q_\alpha, q_\beta\}$ have $sp(2)$ commutation relations

and rotate q_α as a $sp(2)$ vector

$$[T_{\alpha\beta}, T_{\gamma\eta}] = ((\epsilon_{\alpha\gamma}T_{\beta\eta} + \alpha \leftrightarrow \beta) + \gamma \leftrightarrow \eta); \quad [T_{\alpha\beta}, q_\gamma] = \epsilon_{\alpha\gamma}q_\beta + \alpha \leftrightarrow \beta. \quad (3)$$

To describe a doubling of the elementary algebras in $g = hs(2; \nu) \oplus hs(2; \nu)$ it is convenient to introduce an additional central involutive generating element ψ : $[\psi, q_\alpha] = 0$, $[\psi, K] = 0$, $\psi^2 = 1$. The two simple subalgebras of g are singled out by the projection operators $P_\pm = \frac{1}{2}(1 \pm \psi)$.

The full set of HS gauge fields in 2+1 dimensions thus is

$$A(q, K, \psi|x) = dx^\nu \sum_{n=0}^{\infty} \sum_{B=0,1} \frac{1}{n!} (\omega_\nu^{B\alpha_1 \dots \alpha_n}(x) + \psi h_\nu^{B\alpha_1 \dots \alpha_n}(x)) (K)^B q_{\alpha_1} \dots q_{\alpha_n}. \quad (4)$$

The field strengths and gauge transformation laws are defined in the usual way

$$R(q, K, \psi|x) = dA(q, K, \psi|x) + A(q, K, \psi|x) \wedge A(q, K, \psi|x), \quad d = dx^\nu \frac{\partial}{\partial x^\nu};$$

$$\delta A(q, K, \psi|x) = d\epsilon(q, K, \psi|x) + [A(q, K, \psi|x) \wedge \epsilon(q, K, \psi|x)].$$

The gravitational fields $A^{gr} = \frac{1}{2}(\omega^{\alpha\beta} + h^{\alpha\beta}\psi)q_\alpha q_\beta$ take values in the subalgebra $sp(2) \oplus sp(2)$. The pure gauge HS action has the Chern-Simons form. It reduces to the Witten gravity action [3] in the spin 2 sector and to the Blencowe's HS action [4] in the case of $\nu = 0$. An important question is then how to introduce interactions of HS gauge fields with propagating matter fields.

2 Unfolded equations

In this report we will show how this problem can be solved at the level of equations of motion using an approach which we call “unfolded formulation” [8]. It consists of rewriting dynamical equations in a form of certain zero-curvature conditions and covariant constantness conditions

$$d\omega = \omega \wedge \omega, \quad dB^A = \omega^i t_i^A B^B, \quad (5)$$

supplemented with some gauge invariant constraints

$$\chi(B) = 0 \quad (6)$$

which do not contain space-time derivatives. Here $\omega(x) = dx^\nu \omega_\nu^i(x) T_i$ is a gauge field taking values in some Lie superalgebra l ($T_i \in l$), and $B^A(x)$ is a set of 0-forms which take values in a representation space of some representation $(t_i)^B_A$ of l .

An interesting property of this form of equations is that their dynamical content is hidden in the constraints (6). Indeed, locally one can integrate out explicitly (5) as $\omega = dg(x)g^{-1}(x)$ and $B(x) = t_{g(x)}(B_0)$ where $g(x)$ is an arbitrary invertible element while B_0 is an arbitrary x -independent representation element and $t_{g(x)}$ is the exponential of the representation t of l . Since the constraints $\chi(B)$ are gauge invariant one is left with the only condition $\chi(B_0) = 0$. Let $g(x_0) = I$ for some point of space-time x_0 . Then $B_0 = B(x_0)$. One can wonder how any restrictions on values of some 0-forms in a fixed point of space-time can lead to a non-trivial dynamics. This is possible if the set of 0-forms B is rich enough to describe all space-time derivatives of dynamical fields while the constraints (6) just impose all restrictions on the space-time derivatives required by the dynamical equations under consideration. Given solution of (6) one knows all derivatives of the dynamical fields compatible with the field equations and can therefore reconstruct these fields by analyticity in some neighborhood of x_0 . The specificity of the HS dynamics which makes such an approach adequate is that HS symmetries mix all orders of derivatives which therefore have to be contained in the representations t of HS symmetries.

To illustrate this point let us consider an example of a scalar field ϕ obeying the massless Klein-Gordon equation $\square\phi = 0$ in a flat space-time of an arbitrary dimension d . Here l is identified with the Poincare algebra $iso(d-1, 1)$ which gives rise to the gauge fields $\omega_\nu = (h_\nu^a, \omega_\nu^{ab})$ ($a, b = 0 - (d-1)$). The zero curvature conditions of $iso(d-1, 1)$, $R_{\nu\mu}^a = 0$, $R_{\nu\mu}^{ab} = 0$, imply that the vierbein h_ν^a and Lorentz connection ω_ν^{ab} describe the flat geometry. Fixing the local Poincare gauge transformations one can set $h_\nu^a = \delta_\nu^a$, $\omega_\nu^{ab} = 0$.

To describe dynamics of a spin zero massless field $\phi(x)$ let us introduce an infinite collection of 0-forms $\phi_{a_1 \dots a_n}(x)$ which are totally symmetric traceless tensors $\eta^{bc}\phi_{bca_3 \dots a_n} = 0$, where η^{bc} is the flat Minkowski metrics. The “unfolded” version of the Klein-Gordon equation has a form of the following infinite chain of equations

$$\partial_\nu \phi_{a_1 \dots a_n}(x) = h_\nu^b \phi_{a_1 \dots a_n b}(x), \quad (7)$$

where we have replaced the Lorentz covariant derivative by the ordinary flat derivative ∂_ν using the gauge condition $\omega_{\nu,ab} = 0$. The tracelessness condition for ϕ is a specific realization of the constraints (6) while the system of equations (7) is a particular example of the equations (5). It is easy to see that this system is formally consistent, i.e. ∂_μ differentiation of (7) does not lead to new conditions after antisymmetrization $\nu \leftrightarrow \mu$. This property is equivalent to the fact that the set of zero forms $\phi_{a_1 \dots a_n}$ spans some representation of the Poincare algebra.

To show that the system (7) is equivalent to the free massless field equation $\square\phi(x) = 0$ let us identify the scalar field $\phi(x)$ with the $n = 0$ member of the tower of 0-forms $\phi_{a_1 \dots a_n}$. Then the first two equations (7) read $\partial_\nu \phi = \phi_\nu$ and $\partial_\nu \phi_\mu = \phi_{\mu\nu}$,

respectively. The first one tells us that ϕ_ν is a first derivative of ϕ . The second one implies that $\phi_{\nu\mu}$ is a second derivative of ϕ . However, because of the tracelessness condition for $\phi_{\nu\mu}$ it imposes the Klein-Gordon equation $\square\phi = 0$. It is easy to see that all other equations in (7) express highest tensors in terms of the higher-order derivatives $\phi_{\nu_1\dots\nu_n} = \partial_{\nu_1}\dots\partial_{\nu_n}\phi$ and impose no additional conditions on ϕ . The tracelessness conditions are all satisfied once the Klein-Gordon equation is true.

3 Free fields in 2+1 AdS space

Let us now confine ourselves to the 2+1 dimensional case and generalize the above analysis of the scalar field dynamics to the AdS geometry. The gauge fields of the AdS algebra $o(2,2) \sim sp(2) \oplus sp(2)$ are identified with the gravitational fields $A_\nu = (\lambda h_{\nu,\alpha\beta}; \omega_{\nu,\alpha\beta})$. The zero-curvature conditions $R_{\nu\mu} = 0$ for the AdS algebra in its orthogonal realization take a form

$$R_{\nu\mu,ab} = \lambda^2(h_{\nu a}h_{\mu b} - h_{\nu b}h_{\mu a}), \quad R_{\nu\mu,a} = 0 \quad (\nu, \mu \dots; a, b \dots = 0-2), \quad (8)$$

where $R_{\nu\mu,ab}$ and $R_{\nu\mu,a}$ are the Riemann and torsion tensors, respectively. From (8) one concludes that the zero curvature equations for the algebra $o(2,2)$ on a 3d manifold does indeed describe the AdS space provided that $h_\nu{}^a$ is identified with a dreibein and is invertible.

It is an important property of the 3d geometry that one can resolve the tracelessness conditions for symmetric tensor fields by using the formalism of two-component spinors: a totally symmetric traceless tensor $\phi_{\mu_1\dots\mu_n}$ is equivalent to a totally symmetric multispinor $C_{\alpha_1\dots\alpha_{2n}}$. Let us now address the question what is a general form of the equations analogous to (7) such that their integrability conditions reduce to (8). The result is that these are equations of the form [8]

$$DC_{\alpha_1\dots\alpha_{2n}} = h^{\beta\gamma}C_{\alpha_1\dots\alpha_{2n}\beta\gamma} + 2n(2n-1)e(2n, \lambda, M)h_{\{\alpha_1\alpha_2}C_{\alpha_3\dots\alpha_{2n}\}\alpha}, \quad (9)$$

where D is the Lorentz covariant derivative, $DB_\alpha = dB_\alpha + \omega_\alpha{}^\beta B_\beta$, and $e(l, \lambda, M) = \frac{1}{4}\lambda^2 - \frac{1}{2}\frac{M^2}{l^2-1}$, ($l \geq 2$). M is an arbitrary parameter. One can see that this freedom in M is just the freedom of the relativistic field equations in the parameter of mass.

Thus the equations (9) describe a scalar field of an arbitrary mass in 2+1 dimensions. Now let us show how these equations can be generated with the aid of the generalized oscillators (2). To this end we introduce the generating function

$$C(q_\alpha, K|x) = \sum_{A=0,1} \sum_{n=0}^{\infty} \frac{1}{n!} C_{\alpha_1\dots\alpha_n}(x) (K)^A q^{\alpha_1} \dots q^{\alpha_n}. \quad (10)$$

The relevant equations acquire then the following simple form

$$D^L C(q_\alpha, K|x) = \{h^{\alpha\beta} q_\alpha q_\beta, C(q, K|x)\} \quad (11)$$

(from now on we use the dimensionless units with a unit AdS radius, $\lambda = 1$). To see that the integrability conditions for (11) reduce to the zero-curvature conditions for $sp(2) \oplus sp(2)$ one observes that there is an automorphism of the AdS algebra which changes a sign of the AdS translations. This automorphism allows one to introduce a “twisted representation” of the AdS algebra with the anticommutator (instead of the commutator) in the translational part of the AdS algebra. This twisted representation just leads to the covariant constantness equations (11).

Since the part which depends on the background gauge fields in (11) only involves even combinations of the oscillators q_α the full system of equations decomposes into four independent subsystems which can be singled out by virtue of the projection operators $P_\pm = \frac{1}{2}(1 \pm K)$ either in the boson or in the fermion sectors (even (odd) functions $C(q_\alpha, K|x)$ of q_α describe bosons (fermions)). The explicit calculation which involves some reorderings of q_α and rescalings of fields then shows that the irreducible boson subsystems projected out by P_\pm indeed reduce to the equations of motion of the form (9) for a massive scalar field of mass $M^2 = \frac{1}{2}\nu(\nu \mp 2)$. Remarkably, the same equations in the fermion sector describe spin $\frac{1}{2}$ fermion fields of the mass $M^2 = \frac{1}{2}\nu^2$.

An important achievement of the reformulation of the free field equations in the form (11) is that this form suggests that the global HS symmetry algebra realized on the matter fields of mass $M(\nu)$ is $g = hs(2; \nu) \oplus hs(2; \nu)$ with the gauge fields (4). However to simplify the formulation it is convenient to introduce two Clifford variables $\psi_{1,2}$ ($\{\psi_i, \psi_j\} = \delta_{ij}$) instead of ψ . One then introduces the full set of HS gauge fields as $W_\nu(q_\alpha, K, \psi_{1,2}|x)$ and realizes the gravitational fields as

$$W_\nu^{gr} = \frac{1}{2}(\omega_\nu^{\alpha\beta} + h_\nu^{\alpha\beta} \psi_1) q_\alpha q_\beta. \quad (12)$$

The generating function for 0-forms is

$$C(q, K, \psi_{1,2}|x) = C^{mat}(q, K, \psi_1|x) \psi_2 + C^{aux}(q, K, \psi_1|x). \quad (13)$$

Now let us consider the zero curvature equations $0 = R \equiv dW(q, K, \psi|x) + W(q, K, \psi|x) \wedge W(q, K, \psi|x)$ along with the covariant constantness conditions in the adjoint representation of the HS algebra

$$0 = \mathcal{D}C \equiv dC(q, K, \psi|x) + W(q, K, \psi|x)C(q, K, \psi|x) - C(q, K, \psi|x)W(q, K, \psi|x). \quad (14)$$

Due to the factor of ψ_2 in front of C^{mat} the equations for C^{mat} turn out to be equivalent to the equations (11) in the gauge in which only the gravitational part (12) of the set of HS gauge fields is non-vanishing. The fields C^{aux} can be shown [8] to be of a topological type so that each irreducible subsystem in this sector can describe at most a finite number of degrees of freedom and trivializes in a topologically trivial situation. Thus the effect of introducing a second Clifford element consists of addition of some topological fields.

4 Non-linear dynamics

To describe non-linear HS dynamics of matter fields in 2+1 dimensions we start with a system of equations which is very close to that introduced in [9] for a particular case of massless matter fields. We introduce three types of the generating functions $dx^\nu W_\nu(z_\alpha, y_\beta, K, \psi_i|x)$, $s_\gamma(z_\alpha, y_\beta, K, \psi_i|x)$ and $B(z_\alpha, y_\beta, K, \psi_i|x)$ which depend on the space-time variables x^μ and auxiliary variables $(z_\alpha, y_\beta, K, \psi_i)$ such that the two Clifford elements ψ_i commute to all other variables, while the bosonic spinor variables z_α and y_β commute to each other but anticommute with K , $\{K, z_\alpha\} = \{K, y_\alpha\} = 0$, $K^2 = 1$. Their physical content is as follows: $dx^\nu W_\nu$ is the generating function for HS gauge fields, B contains physical matter degrees of freedom along with some auxiliary variables, and s_γ is entirely auxiliary variable which allows one to formulate the full system of equations in a compact form. This formulation is based on the following star-product law which endows the space of functions $f(z, y)$ with a structure of associative algebra

$$(f * g)(z, y) = (2\pi)^{-2} \int d^2u d^2v f(z + u, y + u) g(z - v, y + v) \exp i(u_\alpha v^\alpha). \quad (15)$$

This product law provides a particular symbol realization of the Heisenberg–Weyl algebra, $[y_\alpha, y_\beta]_* = -[z_\alpha, z_\beta]_* = 2i\epsilon_{\alpha\beta}$.

The full system of equations has the form:

$$dW + W * \wedge W = 0, \quad ds_\alpha + \tilde{W} * s_\alpha - s_\alpha * W = 0, \quad dB + W * B - B * W = 0, \quad (16)$$

and

$$\tilde{s}_\alpha * s_\beta - \tilde{s}_\beta * s_\alpha = -2i\epsilon_{\alpha\beta}(1 + \kappa * B), \quad \tilde{B} * s_\alpha - s_\alpha * B = 0, \quad (17)$$

where $\kappa = K \exp i(z_\alpha y^\alpha)$ is a central element of the algebra which has vanishing star commutators with y_α , z_α , K and ψ_i , while $\tilde{a}(z, y, K, \psi_i|x) = a(z, y, -K, \psi_i|x) \forall a$.

The system of equations (16),(17) is explicitly invariant under the general coordinate transformations and the HS gauge transformations $\delta W = d\epsilon + W * \epsilon - \epsilon * W$, $\delta B = B * \epsilon - \epsilon * B$ and $\delta s_\alpha = s_\alpha * \epsilon - \tilde{\epsilon} * s_\alpha$. To elucidate its physical content one has to

analyze this system perturbatively near some vacuum solution. In the massless case the appropriate vacuum solution [9] is $B_0 = 0$, $s_{0\alpha} = z_\alpha$ and $W_0 = \omega(y, K, \psi_{1,2})$ with the vacuum gauge field ω satisfying the zero curvature condition $d\omega + \omega * \wedge \omega = 0$. It can be shown along the lines of [9] that the system of equations (16),(17) expanded near this vacuum solution properly describes dynamics of massless matter fields on the free field level and beyond.

The main result of this report consists of the observation that the same system (16), (17) expanded near another vacuum solution describes dynamics of matter fields with an arbitrary mass. This is a solution with $B_0 = \nu$ where ν is an arbitrary constant. For a constant field B_0 only the first of the equations (17) remains non-trivial. Remarkably it turns out to be possible to find its explicit solution

$$s_{0\alpha} = z_\alpha + \nu(z_\alpha - y_\alpha) \int_0^1 dt t e^{it(z_\alpha y^\alpha)} K \quad (18)$$

(it is not too difficult to check that (18) satisfies (17) by a direct substitution). Now let us turn to the equations (16). The third of these equations is trivially satisfied. The second one reads

$$\tilde{W} * s_{0\alpha} - s_{0\alpha} * W = 0, \quad (19)$$

where we have taken into account that $ds_{0\alpha} = 0$. Eq. (19) is a complicated integral equation. The key observation however is that it admits the following two particular solutions: $W_0 = q_\alpha$ ($\alpha = 1, 2$),

$$q_\alpha = y_\alpha + \nu K(z_\alpha - y_\alpha) \int_0^1 dt (1-t) e^{itz_\alpha y^\alpha}. \quad (20)$$

Taking into account that $*$ -product is associative it allows us to describe a general solution of (19) as an arbitrary element $W_0 = \omega(q_\alpha, K, \psi_{1,2}|x)$ whose arguments are treated as some non-commutative elements of the star-product algebra. To make contact with the previous consideration it remains to check by the explicit computation that the elements q_α indeed obey (2). Thus, the vacuum solution with a constant field $B_0 = \nu$ leads automatically to the deformed oscillator algebra with the deformed oscillators realized as elements of the tensor product of two Heisenberg-Weyl algebras. Finally, it remains to observe that the first of the equations (16) reduces to the zero curvature equation which describes the AdS background space.

Next one can analyze the full system of equations perturbatively by inserting the expansions of the form: $W = W_0 + W_1 + \dots$, $B = B_0 + B_1 + \dots$ and $s_\alpha = s_{0\alpha} + s_{1\alpha} + \dots$. In particular one can derive in the lowest orders that $B_1(z, y, K, \psi|x) = C(q, K, \psi|x)$, $W_1(z, y, K, \psi|x) = \omega(q, K, \psi|x) + \Delta W_1(C)$ and $s_{1\alpha} = s_{1\alpha}(C)$, where $s_{1\alpha}(C)$ and $\Delta W_1(C)$ are some functionals of the field C which remains arbitrary and has to be identified with generating function (13). Inserting this back into (16) one

obtains in the linearized approximation the free field equations for C from the third equation and the equations of the form $d\omega + \omega * \wedge \omega + O(C^2) = 0$ from the first one. Let us note that in the latter case the corrections linear in C are compensated by appropriate field redefinitions so that the sources for HS field strengths (including the gravitational and Yang-Mills ones) acquire in the lowest order a natural structure of some bilinear currents.

5 Concluding remarks

1. It is shown that global HS symmetries depend on the mass of a matter multiplet.
2. The full nonlinear HS model admits continuously degenerate vacua corresponding to a variety of systems with an arbitrary mass of the matter fields. The different global HS symmetries of the linearized matter multiplets are different stability subgroups of the full HS symmetry which leave invariant a chosen vacuum solution.
3. The model under consideration (eq.(11)) possesses $N = 2$ supersymmetry since the deformed oscillator algebra was shown in [10] to contain $osp(2; 2)$ generators $T_{\alpha\beta} = \frac{1}{4i}\{q_\alpha, q_\beta\}$, $Q_\alpha = q_\alpha$, $S_\alpha = q_\alpha K$ and $J = K + \nu$.
4. For special values of the parameter $\nu = \frac{2l+1}{2}$, $\forall l \in \mathbf{Z}$, free field equations degenerate in a certain sense and correspond to particular gauge systems. The origin of this degeneracy is that the algebra $Aq(2; \nu)$ admits ideals for these values of ν [7]. The degenerate point $\nu = 3$ can be shown to correspond to 3d electrodynamics. Interestingly enough, the existence of the ideals of $Aq(2; \nu)$ is related to the existence of a dual version of the model based on a potential for a magnetic field.

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